LEPTON SYMMETRY AND SELF-MASS*

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We consider the symmetry of leptonic weak and electromagnetic interactions unified in a model free of quadratic divergences. Explicit values for intermediate boson masses and coupling constants are obtained from the requirement of finite self-masses for leptons.

With the argument of Lee and Wick[1] that unstable negative-metric states are consistent with unitarity, a new picture has emerged of renormalizable weak interactions mediated by vector bosons (B) together with negative-metric scalar particles (C) of the same mass. Recently Terazawa[2] has considered leptonic weak interactions with this idea. In his model the logarithmic divergences in the lowest order weak and electromagnetic self-masses of leptons cancel, relating the coupling constant of the weak boson to the elementary charge e.

This note considers such a cancellation (which, without some deeper connection between the photon and the weak boson, may appear ad hoc) within a unified model[3,4] of leptonic weak and electromagnetic interactions. In this model the photon and the B bosons are related to the leptons by an SU(2) symmetry. The symmetry group does not connect the electronic leptons ν_e and e to the muonic leptons ν_μ and μ; so it is sufficient to consider the former only. As was pointed out by Glashow,[3] in order to generate the correct electromagnetic current one needs four vector bosons consisting of a triplet and a singlet. The singlet and the neutral member of the triplet mix into the physical photon and a neutral B boson (B^0) forcing the presence of neutral currents in leptonic weak interactions. Since the latter contributes to the weak self-mass of a massive lepton, our results for the coupling constants and masses of B bosons are different from those of Terazawa.[2] In addition we obtain the γ-B^0 mixing angle. Throughout we restrict ourselves to the lowest order since the non-Abelian interaction between the leptons and the vector bosons that we consider is gauge invariant only to the lowest order in the coupling constant.

We can easily construct the nonmass part of the free Lagrangian density as follows:

$$\mathcal{L}_1=-\tfrac{1}{4}(\vec{\mathrm{B}}_{\mu\nu}-g_1\vec{\mathrm{B}}_\mu\times\vec{\mathrm{B}}_\nu)^2-\tfrac{1}{4}B_{\mu\nu}{}^4B^{4\mu\nu}+\tfrac{1}{2}\partial_\mu\vec{\mathrm{C}}\cdot\partial^\mu\vec{\mathrm{C}}+\tfrac{1}{2}\partial_\mu C^4\partial^\mu C^4+i\bar{l}\gamma^\mu\partial_\mu l. \quad (1)$$

In Eq. (1) the vector sign (incorporating indices 1, 2, 3) refers to the triplets and the index 4 to the singlets; $\vec{\mathrm{B}}_{\mu\nu}=\partial_\mu\vec{\mathrm{B}}_\nu-\partial_\nu\vec{\mathrm{B}}_\mu$ and

$$l=\begin{pmatrix}u_\nu\\ u_e{}^L\\ u_e{}^R\end{pmatrix}$$

with $u_e{}^{L,R}=\tfrac{1}{2}(1\mp\gamma_5)u_e$.

The vector-boson–lepton interaction can be obtained by requiring gauge invariance in the lowest order of g_1 under the following transformations:

$$\vec{\mathrm{B}}_\mu\to\vec{\mathrm{B}}_\mu+\partial_\mu\vec{\mathrm{L}}(\kappa)+g_1\vec{\mathrm{L}}(\kappa)\times\vec{\mathrm{B}}_\mu,\quad B_\mu{}^4\to B_\mu{}^4+\partial_\mu K(\kappa),\quad l\to e^{ig_1\vec{\mathrm{W}}\cdot\vec{\mathrm{L}}(\kappa)}e^{ig_2W_4K(\kappa)}l.$$

In the last transformation the matrices $W_{1,2,3,4}$ are those defined by Gell-Mann.[5] This part of the Lagrangian density is then obtained by changing ∂_μ in the lepton piece to $\partial_\mu-ig_1\vec{\mathrm{W}}\cdot\vec{\mathrm{B}}_\mu-ig_2W_4B_\mu{}^4$ to be

$$\mathcal{L}_2=g_1\vec{\mathrm{B}}_\mu\cdot\bar{l}\gamma^\mu\vec{\mathrm{W}}l+g_2B_\mu{}^4\bar{l}\gamma^\mu W^4l. \quad (2)$$

The mass part of the Lagrangian density can be introduced explicitly or can arise out of a spontaneous Goldstone breaking.[4] It is simply

$$\mathcal{L}_3=\tfrac{1}{2}m^2(B_\mu{}^1B^{1\mu}+B_\mu{}^2B^{2\mu})+\tfrac{1}{2}M^2B_\mu{}^0B^{0\mu}+\tfrac{1}{2}m^2(C^1C^1+C^2C^2)+\tfrac{1}{2}M^2C^0C^0. \quad (3)$$

In Eq. (3) the relation between m and M is given by the details of the γ-B^0 mixing. We have to consider these details before we can write down the scalar-boson–lepton interaction.

The physical photon and B^0 fields are given, respectively, by

$$A_\mu=B_\mu{}^3\cos\theta+B_\mu{}^4\sin\theta,\quad B_\mu{}^0=-B_\mu{}^3\sin\theta+B_\mu{}^4\cos\theta. \quad (4)$$

Since the photon is massless and B^0, B^3, and B^4 are ascribed masses M, m, and μ, respectively, the relations among the masses are[6]

$$M^2=m^2+\mu^2, \quad \cot^2\theta=\mu^2/m^2. \tag{5}$$

Equation (2) can now be written in terms of the photon and B^0 fields. However, in order to generate the correct electrodynamics of leptons in terms of the electromagnetic current $e\bar{l}\gamma_\mu(W_3+W_4)l$, we have to require that

$$g_1\cos\theta=g_2\sin\theta=e. \tag{6}$$

Using Eqs. (4) and (6), Eq. (2) can be rewritten as

$$\mathfrak{L}_2=eA^\mu\bar{l}\gamma_\mu(W_3+W_4)l+e\sec\theta\bar{l}\gamma^\mu(B_\mu{}^1W_1+B_\mu{}^2W_2)l+eB_\mu{}^0\bar{l}\gamma^\mu(W_4\cot\theta-W_3\tan\theta)l;$$

or

$$\mathfrak{L}_2=eA^\mu\bar{u}_e\gamma_\mu u_e+\frac{e\sec\theta}{2\sqrt{2}}[B_\mu{}^+\bar{u}_\nu\gamma^\mu(1-\gamma_5)u_e+B_\mu{}^-\bar{u}_e\gamma^\mu(1-\gamma_5)u_\nu]$$
$$-\frac{e}{4}B_\mu{}^0[\bar{u}_\nu\gamma^\mu(1-\gamma_5)u_\nu(\tan\theta+\cot\theta)+\bar{u}_e\gamma^\mu u_e(3\cot\theta-\tan\theta)+\bar{u}_e\gamma^\mu\gamma_5u_e(\cot\theta+\tan\theta)]. \tag{7}$$

In Eq. (7) $B_\mu{}^\pm=(1/\sqrt{2})(B_\mu{}^1\mp iB_\mu{}^2)$. Thus we obtain

$$e^2\sec^2\theta/8m^2=G/\sqrt{2}, \tag{8}$$

where G is the Fermi constant.

The scalar-boson–lepton interaction now has to be constructed so as to make the theory free of quadratic divergences. This part can be written as

$$\mathfrak{L}_4=\frac{ie\sec\theta}{2\sqrt{2}m}[\partial_\mu C^+\bar{u}_\nu\gamma^\mu(1-\gamma_5)u_e+\partial_\mu C^-\bar{u}_e\gamma^\mu(1-\gamma_5)u_\nu]+\frac{ie}{m}\partial_\mu C^0\bar{l}\gamma^\mu(W_4\cot\theta-W_3\tan\theta)l. \tag{9}$$

$\mathfrak{L}_1$, $\mathfrak{L}_2$, $\mathfrak{L}_3$, and $\mathfrak{L}_4$ as given by Eqs (1), (7), (3), and (9), respectively, complete the construction of the Lagrangian density for our purpose. The lowest order lepton self-mass can now be computed and the logarithmic divergence is given by

$$\Delta m_\nu=0, \quad \Delta m_l=\Delta m_l{}^{EM}+\Delta m_l{}^{B^\pm}+\Delta m_l{}^{B^0} \tag{10}$$

for $l\neq\nu$. For Eq. (10)

$$\Delta m_l{}^{EM}=\frac{3e^2}{16\pi^2}m_l\ln\Lambda^2, \quad \Delta m_l{}^{B^\pm}=-\frac{e^2\sec^2\theta}{64\pi^2}m_l\ln\Lambda^2, \quad \Delta m_l{}^{B^0}=\frac{e^2}{64\pi^2}m_l\frac{3(2\cos2\theta+1)^2-5}{\sin^22\theta}\ln\Lambda^2,$$

where Λ is the common cutoff. Hence $\Delta m_{l\neq\nu}=0$ for $\cot^2\theta=3/11$ and $|\theta|=62°4'$. Because of Eq. (8) this means $m=87M_p$ for the charged bosons. On the other hand, Eq. (5) gives $M=98M_p$ for the physical neutral bosons and $\mu=45M_p$. The weak coupling constant of the charged bosons is $\frac{1}{2}e(7/3)^{1/2}$.

In conclusion, we note that our results qualitatively agree (since $|\theta|=62°4'$) with the frequent contention that weak boson coupling strengths are strong enough to compete with electromagnetism. Our weak boson masses agree with the lower bounds of Weinberg.[4] However, Glashow's[3] speculation that the B^0 mass is considerably higher than the $B^\pm$ mass is not borne out.[7] The effect of the weak neutral current should be manifest in $e\nu$ scattering where our $e\nu$ interaction is

$$(G/\sqrt{2})\bar{u}_\nu\gamma_\mu(1-\gamma_5)u_\nu[(15/14)\bar{u}_e\gamma^\mu u_e-\tfrac{3}{2}\bar{u}_e\gamma^\mu\gamma_5u_e],$$

to be compared with

$$-(G/\sqrt{2})\bar{u}_\nu\gamma_\mu(1-\gamma_5)u_\nu\bar{u}_e\gamma^\mu(1-\gamma_5)u_e$$

of the standard theory.[8]

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