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Historical Note on the Derivation of the Strength of Neutral-Current Weak Interactions.

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The Glashow-Weinberg-Salam theory [1-3] of electroweak interactions has been rightly acknowledged [4] recently as a major advance in the understanding of these fundamental interactions. The unification between a short-range force (with apparently uncontrollable infinities in its perturbation theory) and a long-range force (for which the said infinities have been understood and controlled already for over three decades via the renormalization procedure) through the framework of a spontaneously broken non-Abelian gauge theory is one of the greatest theoretical achievements of our time. In terms of experimental confirmation also the strength and structure of the predicted new kind of weak processes—*viz.* those induced by neutral currents—have been brilliantly verified in many laboratories. The same is true of another prediction, namely that of a new type of quark carrying the quantum number « charm ».

The present note concerns an amusing historical aspect regarding the derivation of the strength of the neutral-current weak interaction in the above theory. It may be recalled that historically the first major component of this theory came out in Glashow's 1961 paper [5].

Glashow's model had a $SU_2 \times U_1$ gauge symmetry to start with. However, it was broken by explicit mass terms inserted for the gauge bosons. In principle, these could preserve the global $SU_2 \times U_1$ symmetry. However, the latter should be violated by the

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(1) S. L. Glashow: *Rev. Mod. Phys.*, **52**, 539 (1980) and references therein.

(2) S. Weinberg: *Rev. Mod. Phys.*, **52**, 515 (1980) and references therein.

(3) A. Salam: *Rev. Mod. Phys.*, **52**, 525 (1980) and references therein.

(4) *The Nobel Prize for Physics* (1979).

(5) S. L. Glashow: *Nucl. Phys.*, **22**, 579 (1961); A. Salam and J. C. Ward: *Phys. Lett.*, **13**, 168 (1964). Concerning those two articles, we note: i) the $SU_2 \times U_1$ symmetry is broken through mass term insertions in the lepton-quark sector as well as in the weak-boson one; ii) it is not stated explicitly that the $W^{(3)}$ mass is the same as the $W^{\pm}$ mass, this is done explicitly in ref. (7).

introduction of a mass-mixing term. Thus the neutral SU_2 boson mixed with the U_1 gauge boson (with a weak mixing angle θ_W, say) to yield two orthogonal physical vector bosons: the massive weak neutral Z and the massless electromagnetic γ.

Of course, such explicit mass-insertions were theoretically *ad hoc* and were incompatible with renormalizability. The important contribution of Weinberg (²) and Salam (³) during 1967-68 was to give an explicit mechanism (through an elementary Higgs scalar SU_2-doublet) for this mass generation utilizing the spontaneous-symmetry-breakdown ideas of Higgs, Brout, Englert and Kibble. This preserved renormalizability, as was demonstrated later by 'T HOOFT and others. One consequence of the Weinberg-Salam contribution was the relation $M_Z|\cos\theta_W| = M_W$ linking the masses of the physical charged and neutral weak bosons. Given the universality of the SU_2 gauge couplings, the strength of the neutral current weak interaction was then fixed.

It has been believed by many leading physicists (⁶) that the strength of the neutral-current weak interaction could not be fixed in Glashow's model without the contribution of Weinberg and Salam or something akin to that; in other words, the impression is that—even at a phenomenological level—the Glashow model could not have been a complete description of leptonic (say) weak interactions in the lowest order of perturbation theory. We wish to focus the reader's attention on the fact that this belief is not well founded. The formula $M_Z = M_W|\cos\theta_W|^{-1}$ fixing the neutral-current strength follows immediately (⁷), as soon as one tries to implement the mass insertion and mixing scheme of Glashow in the most natural way. Such a way would be to proceed somewhat in analogy with the electromagnetic mixing of the hadronic vector bosons ρ° and ω in a broken Yang-Mills theory of strong interactions *à la* Sakurai. In other words, first break the local gauge symmetry (but preserving the global one) by inserting suitable vector boson mass terms. Then, break the global symmetry by the mass mixing term. That is all that is required to relate M_W to M_Z.

The fact that this was not explicitly observed by GLASHOW in 1961 was an oversight and a curious historical accident. In fact, searching in the literature, it may be found that the result has been noted only by a handful of authors so far. This was first done by ourselves (⁷) in 1969 and then on the basis of that work by ENGLERT and BROUT (⁸) and by VELTMAN (⁹) in 1974. It was also noted independently by LLEWELLYN-SMITH (¹⁰) in 1973. Later, the observation was made by BJORKEN (¹¹) in 1976 and by

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(⁶) S. WEINBERG: *Rev. Mod. Phys.*, **52**, 515 (1980), third paragraph of the second column on p. 518; S. L. GLASHOW: *Rev. Mod. Phys.*, **52**, 539 (1980), fifth paragraph of the first column on p. 541; *Search and Discovery, Phys. Today*, **32**, 17 (1979), second paragraph of the third column on p. 17; M. VELTMAN: *Proceedings of the VI International Symposium on Electron and Photon Interactions at High Energies*, edited by H. ROLLNIK and W. PFEIL, third paragraph (Amsterdam, 1973), p. 443.

(⁷) J. PESTIEAU and P. ROY: *Phys. Rev. Lett.*, **23**, 349 (1969). The national « dictionary » between this paper and the present one is: $M = M_Z$, $m = M_W$, $\mu = M_B$, $\theta = \pi/2 - \theta_W$. Let us make two points regarding our 1969 Letter. 1) This was *one* publication in 1969 citing Weinberg's 1967 work-contrary to the claim made via the *Science Citation Index* by S. COLEMAN: *Science*, **206**, 1291 (1979), first paragraph, third column, that the number of such citations in that year was zero. 2) An outrageous but tantalizing conclusion from this work is that the requirement of finite self-masses for leptons to one-loop order in the Glashow model employing the 't Hooft-Feynman gauge implies $\text{ctg}^2\theta = 3/11$ or $\sin^2\theta_W = 3/14$ in uncanny agreement with the experimental value (¹³) of 0.218 ± 0.025. For more recent work along this line in the Glashow-Weinberg-Salam model, see R. DECKER and J. PESTIEAU: *Lett. Nuovo Cimento*, **29**, 560 (1980).

(⁸) F. ENGLERT and R. BROUT: *Phys. Lett. B*, **49**, 77 (1974).

(⁹) M. VELTMAN: private communication to J. P. (1974); *Quarks and leptons*, in *Cargèse, 1979*, edited by M. LÉVY, J. L. BASDEVANT, D. SPEISER, J. WEYERS, R. GASTMANS and M. JACOB (New York, N. Y., 1979), p. 1.

(¹⁰) C. H. LLEWELLYN SMITH: in *Phenomenology of Particles at High Energies*, edited by R. L. CRAWFORD and R. JENNINGS (New York, N. Y., 1974), p. 485.

(¹¹) J. D. BJORKEN: *Proceedings of the* SLAC *Summer Institute, 1976*, edited by M. ZIPF, SLAC Report No. 198 (1976), p. 1.

others [12] in 1978 that the relation $M_Z = M_W|\cos\theta_W|^{-1}$ derived from more general principles rather than an explicit Higgs doublet.

However, in view of the prevailing confusion over the issue and with the aim of improving the historical perspective on it, we shall give here a clear derivation of the above result.

The starting $SU_2\times U_1$ gauge-symmetric Lagrangian density of Glashow's model is

$$\mathscr{L}^{(0)} = -\tfrac{1}{4}\boldsymbol{W}_{\mu\nu}\boldsymbol{W}^{\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}\,.$$

Here

$$\boldsymbol{W}_{\mu\nu} = \partial_\mu\boldsymbol{W}_\nu - \partial_\nu\boldsymbol{W}_\mu + g\boldsymbol{W}_\mu\times\boldsymbol{W}_\nu\,,\qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,,$$

with $\boldsymbol{W}_\mu$ as the triplet of Yang-Mills SU_2 gauge fields having g as the coupling constant of self-interaction and with B_μ as the singlet U_1 gauge field. The mass-part inserted in the first step—breaking the gauge but not the global summetry—is

$$\mathscr{L}^{(1)} = \tfrac{1}{2}M_W^2\boldsymbol{W}_\mu\boldsymbol{W}^\mu + \tfrac{1}{2}M_B^2 B_\mu B^\mu\,.$$

The mass mixing term, introduced in the second step and breaking even the global symmetry, is

$$\mathscr{L}^{(2)} = M_{WB}^2 W_\mu^{(3)}B^\mu\,.$$

As already, explained $W_\mu^{(3)}$ and B_μ are not physical fields. The latter are given—for the massive neutral weak boson and the photon respectively—by [13]

$$Z_\mu = W_\mu^{(3)}\cos\theta_W - B_\mu\sin\theta_W\,,\qquad A_\mu = W_\mu^{(3)}\sin\theta_W + B_\mu\cos\theta_W\,.$$

Thus the total mass term can be written in the diagonal form as

$$\mathscr{L}^{(1)} + \mathscr{L}^{(2)} = \tfrac{1}{2}M_W^2\sum_{a=1}^{2}W_\mu^{(a)}W^{\mu(a)} + \tfrac{1}{2}M_Z^2 Z_\mu Z^\mu\,,$$

corresponding to

$$M_\gamma^2 = M_W^2\sin^2\theta_W + M_B^2\cos^2\theta_W + \sin 2\theta_W M_{WB}^2 = 0\,,$$

$$M_Z^2 = M_W^2 + M_B^2\,,\qquad \tfrac{1}{2}(M_W^2 - M_B^2)\sin 2\theta_W + M_{WB}^2\cos 2\theta_W = 0\,.$$

It follows that $\mathrm{tg}^2\theta_W = M_B^2/M_W^2$ or $M_Z|\cos\theta_W| = M_W$. This is given (albeit in different notation) in ref. (7). It is straightforward to see [13] how the strength of the neutral-current interaction is fixed relative to the charged-current one as a consequence.

We conclude with the following comments:

1) It is evident from above that the Glashow model contained a fixed strength for the weak neutral current and a fixed value for the mass of the physical weak neutral

(12) P. Q. HUNG and J. J. SAKURAI: *Nucl. Phys. B*, **143**, 81 (1978), see the discussion after their eq. (55*b*) and on p. 109; H. FRITZSCH: *XIX International Conference on High Energy Physics*, edited by S. HOMMA, M. KAWAGACHI and H. MIYAZAWA (Tokyo, 1978), p. 597; L. M. SEHGAL: private communication.

(13) *Particle Properties Data Booklet* (1980), p. 68.

boson (modulo the weak angle), only the observation had not been explicitly made in 1961.

2) This in no way detracts from the theoretical greatness of the contributions due to WEINBERG (2) and SALAM (3). However, an independent experimental confirmation of their ideas can only come from the observation and study of the predicted physical neutral Higgs boson. The latter fact has already been emphasized by HUNG and SAKURAI (12).

3) In the context of the Higgs mechanism, it is known (14) that the choice of certain higher Higgs representations violates the $M_Z|\cos\theta_W| = M_W$ relations. However, such a choice is incompatible with the pattern of symmetry breaking through the mass-terms of the Glashow model. The Higgs doublet of Weinberg and Salam (or any number (14,15) of these doublets) yields an explicit realization of this pattern.

4) In the context of a dynamically broken symmetry (as opposed to that induced by elementary scalar fields), it has been realized (15) that the $M_Z = M_W|\cos\theta_W|^{-1}$ relation follows more generally form a pattern of symmetry breakdown preserving the weak $|\Delta I| = \frac{1}{2}$ rule, rather than specifically from elementary Higgs doublets. However, that it followed directly from Glashow's original model is a different point.

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(14) D. A. ROSS and M. VELTMAN: *Nucl. Phys. B*, **95**, 135 (1975).
(15) S. WEINBERG: *Phys. Rev. D*, **19**, 1277 (1979); L. SUSSKIND: *Phys. Rev. D*, **20**, 2619 (1979); A. CARTER and A. PAGELS: *Phys. Rev. Lett.*, **43**, 1845 (1979); P. SIKIVIE, L. SUSSKIND, M. VOLOSHIN and V. ZAKHAROV: *Nucl. Phys. B*, **173**, 189 (1980).

Erratum : p. 626, Ref (7) : replace *national* by *notational.*

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